

The Resonance Scattering Phenomenon of Fast Negatively Charged Particles in a Single Crystal *

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 (Dated: March, 1979)

The energy spectrum of the extended attractive potential of a crystallographic row for negatively charged particles has quasi-bound states. It follows that a negatively charged particle with small transversal momentum component ($p_{\perp}R \ll 1$) may undergo resonance scattering. Thus the resonance scattering phenomenon can be observed in a single crystal, when fast electrons move with a small glancing angle ($\theta_0 \ll 1/pR$) to a crystallographic axis. The calculated results for the electrons and angular widths of resonance peaks are consistent with experimental data.

PACS numbers: 61.80.Fe

Usually the resonance scattering phenomenon in elastic collisions is supposed to take place for slow particles, where $pR \ll 1$ (p is the incident particle momentum and R is the potential radius). In this work it is shown that fast ($pR \gg 1$) negatively charged particles scattered by the extended attractive potential of a crystallographic row exhibit the resonance nature. This happens because the energy spectrum of the transversal potential of a crystallographic row has quasi-bound states. The importance of the quasi-bound states was pointed out in the first article on the scattering of fast electrons in a single crystal [1]. The next works [2 - 4] contained efforts to develop this idea.

However only in [5] it was observed that the scattering process has a resonance nature. The observation condition of resonance scattering in this case consists in the slow motion for the transverse component of incident particles ($p_{\perp}R \ll 1$, where $p_{\perp} = p\theta_0$, with θ_0 the glancing angle with respect to the crystallographic axis).

The wave function of a fast particle satisfies the Dirac equation

$$(\Delta + E^2 - M^2 - 2EU)\Psi = 0, \quad (1)$$

where M, E are the mass and energy of the particle, U is external potential ($E \gg U$). For simplicity spin effects will be neglected. The exact wave function of particle in the continuum potential of a crystallographic row with longitudinal length L and transverse radius R ($L > R$) is

$$\Psi = \begin{cases} \Psi_1, & -\infty < z \leq 0, \\ \Psi_2, & 0 \leq z \leq L, \\ \Psi_3, & L \leq z < \infty. \end{cases} \quad (2)$$

Ψ_1, Ψ_3 are the wave functions outside the interaction field which consist of a plane wave and a spherical divergent

wave. Ψ_2 is the wave function inside the interaction field, which we decompose in the set of eigenfunctions in the potential U . It has been assumed that the potential U does not depend on the coordinate z [$U \equiv U(\vec{\rho})$] and the direction of the incident particle momentum is nearly parallel to the axis z . Then Ψ_2 can be written

$$\Psi_2 = \sum_{\alpha, m} Q_{\vec{p}_{\perp}}(\alpha, m) e^{im\phi} Z_{\alpha, m}(\rho) e^{iz\sqrt{p^2 - \alpha}}, \quad (3)$$

where $\exp(im\phi)Z_{\alpha, m}(\rho)$ is the eigenfunction of the particle transverse motion, which satisfies the equation

$$\left[\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} - 2EU(\rho) \right] e^{im\phi} Z_{\alpha, m}(\rho) = -\alpha e^{im\phi} Z_{\alpha, m}(\rho). \quad (4)$$

Using the asymptotic behaviour for the wave function (2) and the continuity conditions on the boundaries $Z = 0$ and $Z = L$, we obtain the scattering amplitude [6]

$$f(\theta, \phi) = \frac{p}{2\pi i} \sum Q_{\vec{p}_{\perp i}}(\alpha, m) Q_{\vec{p}_{\perp f}}^*(\alpha, m) [\exp(i \frac{\alpha - p_{\perp}^2}{2p} L) - 1], \quad (5)$$

where $\vec{p}_{\perp i}$ and $\vec{p}_{\perp f}$ are the transverse components of the initial and final momenta,

$$Q_{p_{\perp}}(\alpha, m) = \int d^2 \vec{\rho} Z_{\alpha, m}(\vec{\rho}) \exp(i \vec{p}_{\perp} \vec{\rho}). \quad (6)$$

One can show that the coefficient $Q_{p_{\perp}}(\alpha, m)$ in an arbitrary potential is

$$Q_{p_{\perp}}(\alpha, m) = \delta(\vec{p}_{\perp} - \vec{p}_{\alpha}) - \frac{f(\vec{p}_{\perp}, \alpha, m)}{p_{\perp}^2 - p_{\alpha}^2 - i\delta},$$

$$p_{\alpha}^2 = \alpha \quad \text{for} \quad \alpha > 0, \quad (7)$$

and

$$Q_{p_{\perp}}(\alpha, m) = -\vec{p}_{\alpha} - \frac{f(\vec{p}_{\perp}, \alpha, m)}{p_{\perp}^2 + p_{\alpha}^2},$$

$$p_{\alpha}^2 = -\alpha \quad \text{for} \quad \alpha < 0. \quad (8)$$

*Nuclear Instruments and Methods, 1980, Vol. 170, No.1-3, pp. 25-26

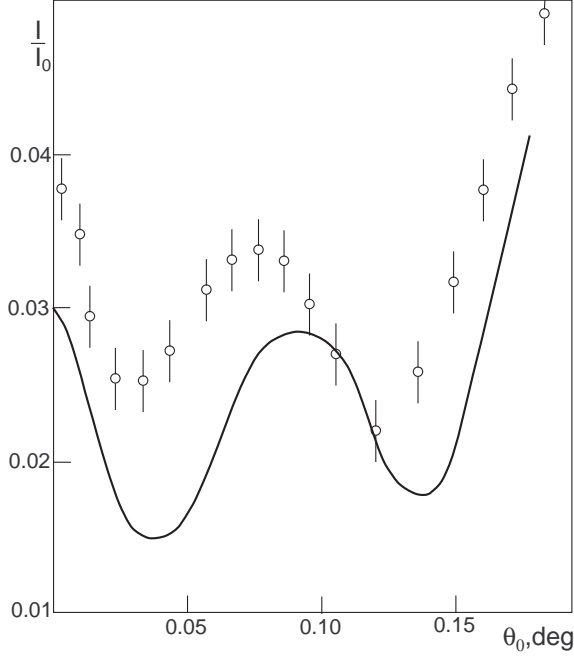


FIG. 1: The solid curve is the relative intensity of the scattered electrons ($E = 15\text{MeV}$) by a single crystal Si ($L = 1.4\mu$) depending on the glancing angle θ_0 with respect the crystallographic axis (111). The experimental data are taken from [7].

Here the function $f(\vec{p}_\perp, \alpha, m)$ is given by

$$f(\vec{p}_\perp, \alpha, m) = 2E \int d^2 \vec{\rho} e^{i\vec{p}_\perp \cdot \vec{\rho}} U(\vec{\rho}) e^{-im\phi} Z_{\alpha, m}(\vec{\rho}). \quad (9)$$

If $\alpha = p_\perp^2$, the function $f(\vec{p}_\perp, \alpha, m)$ is the two-dimensional scattering amplitude by the potential $U(\rho)$. Using the optical theorem $\sigma_{tot} = (4\pi/p) \text{Im}[f(\theta_0, \phi_0)]$, we have

$$\sigma_{tot} = 4 \sum_{\alpha, m} \left| \frac{f(\vec{p}_\perp, \alpha, m)}{p_\perp^2 - \alpha} \right|^2 \sin^2\left(\frac{p_\perp^2 - \alpha}{4p} L\right). \quad (10)$$

As $f(\vec{p}_\perp, \alpha, m)$ has a resonance nature in an attractive potential for slow transverse motion ($p_\perp < 1$), the total cross section of the fast particles is represented by the Breit Wigner formula [5]

$$\sigma \simeq \pi \frac{L}{p} \frac{\frac{1}{4}\Gamma^2}{(\theta_0 - \theta_{0res})^2 + \frac{1}{4}\Gamma^2}, \quad (11)$$

where

$$\theta_{0res} \simeq \left(\frac{\pi^2 n^2 + m^2}{E^2 R^2} - 2 \frac{V_0}{E} \right)^{1/2} \quad (12)$$

$$\Gamma \simeq \theta_{0res} \cdot \exp[-|2EV_0 R^2 - \pi^2 n^2|^{1/2}]. \quad (13)$$

In fig. 1 the dependence of the scattering of 15MeV electrons in a silicon crystal with thickness 1.4μ upon the tilt angle is shown. For silicon the potential parameters are $U_0 = 23\text{eV}$, $R^{-1} = me^2 Z^{1/3} = 9.0 \cdot 10^3 \text{eV}$ and we have the narrow central peak $\theta_0 \simeq 0.02^\circ$ [5]. For the lateral peak we get $\theta_{0res} \simeq 0.1^\circ$ and $\Gamma \simeq 0.5 \cdot \theta_{0res}$. Thus the solid curve gives satisfactory agreement with the experimental data (see Fig. 1).

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